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REVIEW

DONALD GILLIES
Philosophical Theories of Probability
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As is often remarked, the concept of probability is mathematically straightforward but philosophically puzzling: there is near-universal consensus about the formal rules that govern probabilities, but little agreement about what these rules actually mean, i.e. about how to *interpret* the formulae of the probability calculus. Donald Gillies' recent book offers a comprehensive examination of the various philosophical accounts of probability that have been developed in the 350 years since the dawn of probability theory. The book is not primarily expository though: the difficulties faced by the interpretations of probability he discusses lead Gillies to his own, quite original account of this elusive concept.

The book's order of presentation is roughly chronological, dealing in turn with the classical theory of Laplace, the logical theories of Keynes and Carnap, the subjective theories of Ramsey and de Finetti, the frequency theory of von Mises, and the propensity theories of Karl Popper and his successors. Gillies accepts the standard view that the classical and logical theories are untenable because of the contradictions to which the principle of indifference gives rise; he considers and rejects the standard ways of trying to salvage the indifference principle by appeal to invariance principles, 'indivisible alternatives' etc. His discussion of the classical and logical theories contains nothing that is essentially novel, but is nonetheless valuable both for its lucidity and its close attention to historical detail, virtues which permeate the whole book.

The failure of the logical theory leads naturally to the subjective theory, which takes a statement's probability to be a particular individual's degree of belief in that statement, and construes the axioms of probability as rationality constraints on an individual's degrees of belief. Gillies offers a detailed examination of all the main topics in subjective probability theory, including the Ramsey-de Finetti (Dutch-book) theorem of which he gives an elegant proof; the issue of countable versus finite additivity, and the relationship between exchangeability and independence. He is broadly sympathetic to the subjectivist approach, arguing that the (synchronic) Dutch-book argument succeeds, and shows the subjective interpretation to be *a* valid interpretation of the mathematical calculus, though not the only one. (Gillies does not discuss the diachronic Dutch-book argument.) However he does not accept de Finetti's attempt to replace the concept of independence with its 'subjective equivalent', exchangeability; on the contrary, we are only justified in regarding a sequence of events as exchangeable if we already know they are independent, he argues.

Despite his sympathy for the subjective theory, Gillies is not a fan of orthodox Bayesianism. He does not regard Bayesian conditionalization as a useful way to model learning from experience or inductive inference, and defends orthodox statistical methods based on testing (unlike Bayesians such as Howson and Urbach [1989], who argue that orthodox statistical methods should be abandoned). He is thus led to seek an objective interpretation of probability which makes sense of standard statistical practice, to complement the subjective interpretation.

Gillies' discussion of the objective probability theories (the frequency theory and various versions of the propensity theory) forms the core of the book, and contains a wealth of original and interesting ideas. His starting point is von Mises' frequency theory, of which he gives a detailed and very useful exposition. Unlike proponents of the classical and logical theories, von Mises regarded probability theory as an *empirical* scientific theory, on a par with other empirical theories such as mechanics; the subject matter of probability theory was sequences of 'repetitive events' such as coin-tossing, which formed what von Mises called 'collectives'. According to von Mises, two empirical laws can be observed to hold true for collectives: first, the relative frequency of any given attribute in a collective (e.g. the attribute 'heads' in a long sequence of coin-tosses) tends towards a fixed value, as more and more members of the collective are considered. Second, collectives are characterised by a lack of order, or are *random*. Von Mises' aim was to obtain the axioms of probability from these two empirical laws by idealisation, which involved replacing finite empirical collectives with infinite mathematical collectives. This led him to his famous 'limiting frequency' definition of probability: the probability of an attribute in a collective is the relative frequency of that attribute in the infinite limit.

Gillies' discussion of von Mises focuses on both technical and philosophical issues. The former include the adequacy of von Mises' notion of randomness, the relationship of his axioms to the standard Kolmogorov axioms, and the status of countable additivity in his theory. The latter include von Mises' conception of probability theory as an empirical science to be derived by abstraction from observed laws, and the philosophy of science which underpinned this conception. Gillies emphasises von Mises' admiration for the operationalist and positivist views of Ernst Mach. Indeed, he argues that the limiting frequency definition of probability was intended by von Mises as an operational definition of a theoretical concept (probability) in terms of an observable quantity (relative frequency), precisely analogous to Mach's attempted operational definition of the concept of Newtonian mass in *The Science of Mechanics*. Gillies' dislike of operationalism is the major source of his dissatisfaction with von Mises' approach, though he remains sympathetic to its spirit, and retains substantial frequentist elements in his own theory.

The propensity interpretation of probability was introduced by Popper to deal with so-called 'single-case' probabilities, i.e. probabilities of *token* events or outcomes. Such probabilities cannot be accommodated in a frequentist framework, where probabilities are always relative to a sequence of repeated events (i.e. a collective), and thus apply to event types, not tokens. Von Mises did not see this as a shortcoming, for he thought that questions about the objective probability of token events made little sense. But Popper thought that objective single-case probabilities were required for the interpretation of quantum mechanics, and developed the propensity theory to this end. His original idea was to treat the probability of an outcome given a set of conditions as the tendency or propensity of those conditions to produce an outcome of that type, *irrespective of whether those conditions are repeated a large number of times or not*. Probability is thus identified with the disposition to produce relative frequencies on repetition of the conditions, rather than with the relative frequencies themselves, as on von Mises' view. Popper later modified his position so that propensities attached not to (repeatable though perhaps unrepeated) conditions, but rather to the complete state of the universe at a time—the relevant propensity being the propensity of the universe to produce a particular outcome on a specific occasion. Gillies calls theories of the first sort 'long-run' propensity theories and those of the second sort 'single-case' propensity theories. (Though as he notes, the first type of propensity theory was meant to cope with single-case probabilities too.)

Gillies' own preference is for a 'long-run' propensity theory, closer to Popper's earlier position than to his later one. His motivation for this position has nothing to do with wanting objective single-case probabilities, however. Indeed he endorses the view, defended by Howson and Urbach among others, that single-case probabilities are always *subjective*. This view allows that probabilities for a token event can be and often are *based* on knowledge of objective probabilities (relative frequencies), but holds that they are themselves simply subjective degrees of belief. So there simply is no objective fact about the probability that the sun will shine tomorrow. Gillies defends this view against various objections, and discusses some of the problems associated with trying to rationally base single-case subjective probabilities on knowledge of frequencies, such as the problem of the relevant reference class etc.

But why, if he eschews objective single-case probabilities, does Gillies want a propensity theory at all? The answer lies in his anti-operationalism. In effect, Gillies regards the long-run propensity theory as a kind of de-operationalized version of the frequency theory, for it treats relative frequencies as *evidence* for propensities and thus probabilities, rather than as part of probability's definition. He offers a general critique of the operationalist idea that theoretical concepts should be definable in observable terms, and proposes in its place an alternative, more liberal view of how theoretical concepts acquire their empirical significance, using the example of the concept of mass in Newtonian mechanics. Gillies argues that the term 'mass' was initially introduced as an undefined primitive, and acquired empirical meaning not through an operational definition, but rather through the assumption that a planet's mass is negligible compared to that of the sun. *Modulo* this assumption, Newton's theory was able to explain Galileo's and Kelper's laws while also rendering them more precise. A closely analogous situation holds in probability theory, he argues. Here the link between the theoretical concept (probability) and the observable one (frequency) is forged not by operational definition but by what Gillies calls a 'falsifying rule for probability statements'—which says in effect that a statistical hypothesis should be regarded as falsified if the test statistic lies in the designated rejection region. *Modulo* this assumption, results about frequencies can be derived from probabilistic hypotheses; specifically, the empirical laws concerning collectives which von Mises noted can be derived from the probability axioms. Thus Gillies' version of the propensity theory endorses von Mises' view that probability theory is an empirical science dealing with sequences of repetitive events, a 'mathematical science of randomness' as he puts it. Our reason for accepting the theory is its ability to explain a host of empirical phenomena concerning such sequences, amassed over the centuries by gamblers among others, and codified in von Mises' empirical laws.

The last interpretation of probability Gillies discusses is the 'intersubjective' theory, which is essentially an extension of subjective probability theory to cover *groups* of individuals. Gillies offers an ingenious extension of the Dutch-book argument to 'group beliefs', and argues that intersubjective probability is the more appropriate concept where there is a high degree of consensus within a social group or scientific community concerning some matter.

Gillies' considered view of probability is thus pluralist. He acknowledges three currently viable interpretations of probability: the subjective, the inter-subjective, and the long-run propensity account explained above. This immediately raises the question: how do the three interpretations relate to one another? The relation between the subjective and inter-subjective concepts is straightforward, but what about the relation between the subjective and long-run propensity concepts, and more generally between subjective and objective probabilities? Gillies acknowledges that this is a 'key problem', but does not say a huge amount about it. (It is somewhat surprising that he does not discuss David Lewis' very influential ideas on this issue.) He claims that in situations to which the long-run propensity concept is applicable, e.g. games of chance, knowledge of objective propensities will often 'induce' agents to set their subjective degree of belief on a particular outcome equal to the objective propensity of that (type of) outcome. But Gillies does not say whether he thinks this is merely a brute psychological fact or rather something that rationality constrains us to do, and he admits that additional background information will sometimes lead agents to have degrees of belief which do *not* equal the corresponding propensities. This last point is surely correct, but the notion of 'additional background information' then cries out for analysis, which Gillies does not provide. This criticism is perhaps slightly unfair, as any theory must treat some notions as primitive. But those who hope that philosophical treatments of probability will shed light on the problems of inductive inference will be somewhat disappointed by Gillies' brevity here. And those of the Bayesian creed will see a vindication of their view that prior probability assignments are ineliminable.

In the final chapter, Gillies tries to reinforce his pluralism by arguing that an objective concept of probability is required in the natural sciences, but a subjective one in the social sciences. This claim is based on two further ones: (i) that the subject matter of the social sciences does not allow the 'repeatable conditions' that are essential to the applicability of the long-run propensity concept, and (ii) that operationalism is appropriate for concepts in the social sciences but not in the natural sciences. (The relevance of this last claim is that propensities are not operationally definable, as we have seen, but subjective probabilities are—they are defined in terms of agents' betting behaviour.) In my view Gillies makes a good case for (i) but not for (ii). Indeed (ii) does not strike me as very plausible. Some concepts in the natural sciences clearly *are* operationally definable, for example the concept of heritability in quantitative genetics. Conversely, some concepts in the social sciences are not, or at least not obviously, operationally definable, for example the concept of alienation in Marxist theory. Gillies' thesis about the need for different probability concepts in the natural and the social sciences may be correct, but his defence of it is not entirely convincing.

These critical remarks notwithstanding, Gillies has written an excellent book. Within a relatively short space, he outlines and assesses virtually all of the major philosophical ideas about probability, and develops an interesting and original position of his own. Owing to its judicious mix of exposition and original argumentation, the book will be of interest to both specialist and student alike. Furthermore, it will make an excellent teaching aid for upper-undergraduate and postgraduate level courses on probability: the prose style is exceptionally lucid, and the mathematically more demanding material is carefully restricted to starred sections in the text. The book is testimony to the fact that the philosophy of probability is alive and well.

References

Howson, C., and Urbach, P. [1989]: *Scientific Reasoning: the Bayesian Approach*, La Salle, Illinois: Open Court.